



LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – MATHEMATICS

SECOND SEMESTER – APRIL 2025

PMT2MC01 – ALGEBRA



Date: 24-04-2025

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

	Answer ALL the questions	(5 x 1 = 5)
1	Answer the following	
a)	Write the center of Z_5 .	
b)	Define external direct product.	
c)	Find the content of integer monic polynomial.	
d)	Give an example for an infinite field extension.	
e)	Define solvable group.	

SECTION A – K2 (CO1)

	Answer ALL the questions	(5 x 1 = 5)
2	MCQ	
a)	Class equation of Z_4 is i) $3 + 1$ ii) $2 + 2$ iii) $1 + 1 + 1 + 1$ iv) $2 + 1 + 1$	
b)	Order of a 3– Sylow subgroup of $Z_{90} \times Z_{15}$ i) 3 ii) 81 iii) 30 iv) 27	
c)	$Z_7 \times Z_7$ is i) Ring b) integral domain c) PID d) field	
d)	The value of $[Q(\sqrt{2}) : Q]$ is i) 3 ii) 2 iii) 4 iv) 8	
e)	Every finite integral domain is a i) field ii) trivial iii) vector space iv) does not exist	

SECTION B – K3 (CO2)

	Answer any THREE of the following	(3 x 10 = 30)
3	Prove that any group with order p^2 is abelian. Also give an example to show that this result need not be true when the order of the group is p^3 .	
4	State isomorphism theorem for direct products.	
5	Show that any group of order 72 is simple.	
6	State and prove division algorithm for polynomials.	
7	Show that $Q(\sqrt{2} + \sqrt{3}) = Q(\sqrt{2}, \sqrt{3})$.	

SECTION C – K4 (CO3)

	Answer any TWO of the following	(2 x 12.5 = 25)
8	Write about Cauchy's theorem with an illustration.	
9	Find conjugate classes and normalizer of all elements of the quaternion group.	
10	State and prove Eisenstein's criterion. Also show that the converse of the statement need not be true.	
11	Prove that for any prime number p and any positive integer n there exists a field with p^n elements.	

SECTION D – K5 (CO4)

	Answer any ONE of the following (1 x 15 = 15)
12	Analyze the relation between partitions of n and the Conjugate classes of S_n . Also write conjugate classes and class equation of S_4 .
13	If L is a finite extension of F and K is a subfield of L containing F then prove that $[K : F]$ divides $[L : F]$. Also give an example.

SECTION E – K6 (CO5)

	Answer any ONE of the following (1 x 20 = 20)
14	Discuss about second and third part of Sylow's theorems. Also write the properties of the group with order $11^2 13^2$
15	Analyze about fundamental theorem of Galois theory

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